

A DPL model of photo-thermal interaction in an infinite semiconductor material containing a spherical hole

Aatef D. Hobiny^{1,a} and Ibrahim A. Abbas^{1,2,b}

¹ Nonlinear Analysis and Applied Mathematics Research Group (NAAM), Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia

² Department of Mathematics, Faculty of Science, Sohag University, Sohag, Egypt

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Abstract. The dual phase lag (DPL) heat transfer model is applied to study the photo-thermal interaction in an infinite semiconductor medium containing a spherical hole. The inner surface of the cavity was traction free and loaded thermally by pulse heat flux. By using the eigenvalue approach methodology and Laplace's transform, the physical variable solutions are obtained analytically. The numerical computations for the silicon-like semiconductor material are obtained. The comparison among the theories, *i.e.*, dual phase lag (DPL), Lord and Shulman's (LS) and the classically coupled thermoelastic (CT) theory is presented graphically. The results further show that the analytical scheme can overcome mathematical problems by analyzing these problems.

1 Introduction

During the last twenty-five years, much effort has been done to investigate the structure of microelectronic and semiconductors through photo-acoustic (PA) and photo-thermal (PT) technology. [1,2] recently, both PA and PT technologies are considered as standard modes which are highly sensitive to photo-excited carrier dynamics. According to Mandelis and Hess [3], an absorption laser beam with modulated intensity leads to the generation photo-carriers, namely, electron-hole pairs; *i.e.*, the carrier-diffusion wave or plasma wave, play a dominant role in the experiments of PA and PT for most semiconductors. Both the thermal and elastic waves are produced as a contribution of the plasma waves depth-dependence that generates the periodic heat and mechanical vacillations. The thermoelastic (TE) mechanism of elastic wave generation is interpreted as the result of the propagation of elastic vacillations towards the material surface due to the thermal waves in that material. In general, this mechanism (TE) depends on the heat generated in the material, which may generate an elastic wave due to thermal expansion and bend which, in turn, produces a quantity of heat, *i.e.*, thermoelastic coupling. The electronic distortion (ED) is defined as periodic elastic deformations in the material due to photo-excited carriers.

Two generalized thermoelasticity theories are well established and well investigated. Replacing the classical Fourier law by postulating a new law of heat conduction, the generalized thermoelastic theory with one relaxation time has been proposed by Lord and Shulman [4]. Green and Lindsay [5] introduced the generalized thermoelasticity theory with two relaxation times. Sherief and Dhaliwal [6] extended the theories of generalized thermoelasticity for an anisotropic body. Tzou [7] suggested the model of dual phase lag (DPL), which describes the interactions between electrons and photons at the microscopic level. On the macroscopic scale, the retarding sources cause delayed responses. For macroscopic formulation, it would be convenient to use the DPL model for investigation of the microstructural effect on the behavior of heat transfer. By the experimental results, the applicability and the physical meanings of the (DPL) model have been supported as in Tzou [8]. Abbas and Zenkour [9] investigated the interactions in a semi-infinite thermoelastic medium under the dual phase lag model due to a ramp-type heating. Abouelregal and Abo-Dahab [10] studied the effects of the dual phase lag model on the magneto-thermoelastic interaction in an infinite non-homogeneous solid having a spherical cavity. On the other hand, Song *et al.* [11,12] studied in detail the generalized thermoelastic vibrations due

^a e-mail: ahobany@kau.edu.sa

^b e-mail: ibrabbas7@yahoo.com (corresponding author)

to optically excited semiconducting microcantilevers. They concluded that the waves reflect in a semiconductor plane under photo-thermal and generalized thermoelasticity theories [13,14].

The various effects of electronic deformation and thermoelasticity in a semiconductor material with uncoupled system equations on the thermal, elastic and plasma characteristics have been studied by many authors [15–17]. Opsal and Rosencwaig [18] presented their study on a semiconducting material based on the results shown by Rosencwaig *et al.* [19]. Abbas [20] investigated the dual phase lag model on photo-thermal interactions in an unbounded semiconducting medium with a cylindrical hole. Hobiny and Abbas [21] studied the photo-thermal wave in an infinite semiconductor material with a cylindrical hole. The eigenvalues approach in Laplace's domain give exact solutions without any restrictions on the actual physical quantity assumptions. Recently, the authors of [22–30] investigated wave propagation in thermoelastic problems.

The main object of the present paper is to study the photo-thermoelastic interaction in an unbounded semiconductor medium containing a spherical hole under the dual phase lag model. By using the approach of eigenvalue and Laplace's transform, the governing non-dimensional equations are processed by using an analytical-numerical technique. The calculations of numerical solutions were made considering a silicon-like semiconducting medium, and the results are verified numerically and represented graphically.

2 Mathematical formulation

The governing equations in the context of the dual phase lag model for the heat conduction equations, the plasma, and motion and can be written as [4,7,31,32]

$$K \left(1 + t_T \frac{\partial}{\partial t} \right) \Theta_{,jj} = -\frac{E_g}{\tau} N + \left(1 + t_q \frac{\partial}{\partial t} + m \frac{t_q^2}{2} \frac{\partial^2}{\partial t^2} \right) \left(\rho c_e \frac{\partial \Theta}{\partial t} + \gamma_t T_o \frac{\partial u_{j,j}}{\partial t} \right), \quad (1)$$

$$D_e N_{,jj} = \frac{\partial N}{\partial t} + \frac{N}{\tau} - k \frac{\Theta}{\tau}, \quad (2)$$

$$(\lambda + \mu) u_{j,ij} + \mu u_{i,jj} - \gamma_n N_{,i} - \gamma_t \Theta_{,i} = \rho \frac{\partial^2 u_i}{\partial t^2}. \quad (3)$$

The constitutive equations are expressed as

$$\sigma_{ij} = (\lambda u_{k,k} - \gamma_n N - \gamma_t \Theta) \delta_{ij} + \mu (u_{i,j} + u_{j,i}). \quad (4)$$

There are three cases:

i) the dual phase lag (DPL) model,

$$0 < t_T < t_q, \quad m = 1;$$

ii) the Lord and Shulman's (LS) theory,

$$t_q = t_o > 0, \quad t_T = m = 0;$$

iii) the classical dynamical coupled theory (CT),

$$t_T = t_q = m = 0,$$

where $N = n - n_o$, n_o is the equilibrium carrier concentration, $\Theta = T - T_o$ is the temperature increment, T_o is the reference temperature, u_i are the displacement components, t_q and t_T denote the finite times required for effective collisions to take place between the photons and electrons and the thermal equilibrium to be obtained, respectively. Tzou called the phase lag of the heat flux for the delay time t_q and the phase lag of the temperature gradient for the other delay time t_T . t_o is the thermal relaxation time (for semiconductor $t_o = 10^{-12}$ – 10^{-10} s), c_e is the specific heat at constant strain, K is the thermal conductivity, ρ is the medium density, σ_{ij} are the components of stress, $\gamma_t = (3\lambda + 2\mu)\alpha_t$, α_t is the linear thermal expansion coefficient, D_e is the carrier diffusion coefficient, λ , μ are the Lamé's constants, τ is the photo-generated carrier lifetime, $\delta_E = E - E_g$, E_g is the energy gap of the semiconductor, E is the excitation energy, $\gamma_n = (3\lambda + 2\mu)d_n$, d_n is the electronic deformation coefficient and $k = \frac{\partial n_o}{\partial \Theta}$ is the coupling parameter of thermal activation [33], t is the time, \mathbf{r} is the position vector and $i, j, k = r, \theta, \phi$ for spherical coordinates.